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Is the electric charge conserved in brane world?

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Abstract: We discuss whether electric charge conservation may not hold in four-dimensional world in models with infinite extra dimensions, i.e. whether escape of charged particles from our brane is consistent with effectively four-dimensional electrodynamics on the brane. We introduce a setup with photon localized on the brane and show that charge leakage into extra dimension is allowed within this setup. The electric field induced on the brane by escaping charge does not obey four-dimensional Maxwell's equations; this field gradually disappears in a causal way. We also speculate on the possibility of the escape of colored particles and formation of colorless free quark states on the brane.

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JHEP08(2000)041

Contents

The discussion of whether electric charge may not be exactly conserved has long history [1]–[4]. In four-dimensional theories, even tiny non-conservation of electric charge leads to contradictions to low-energy tests of quantum electrodynamics unless exotic millicharged particles are introduced [3] (see, however, ref. [5]). A new perspective emerges in theories describing our world as a brane embedded in higher-dimensional space with infinite extra dimensions [6]–[9]. It is conceivable that in these theories, particles initially residing on our brane may eventually leave the brane and disappear into extra dimensions. In fact, this leakage of particles has been found to be generic at least in a class of field theoretic models of localization of matter on a brane: once matter fields get small but non-vanishing masses, the localization becomes incomplete and particles tunnel from the brane into extra dimensions [10].

If particles that leave our brane are electrically charged, their disappearance into extra dimensions would result in the non-conservation of electric charge, as seen by a four-dimensional observer. Of course, in this scenario electric charge is conserved in the full multi-dimensional space; the charge non-conservation in four-dimensional world is merely a consequence of incapability of our devices to detect the charges outside the brane. Still, this scenario has distinctive experimental signatures such as literally disappearing electrons. Hence, an observation of processes like $\mathrm{e}^- \to \textit{nothing}$ would be a strong evidence for the existence of infinite extra dimensions. In a sence, this probe of extra dimensions is complementary to searches expoloiting the possibility of low fundamental gravity scale: the latter possibility exists irrespectively of whether extra dimensions are compact [11, 12, 13] or infinite [14] whereas the disappearance of matter emerges in theories with infinite extra dimensions but does not require the low fundamental scale of gravity. The rates of proccesses like $\mathrm{e}^- \to \textit{nothing}$ are naturally small [10] but presently cannot be reliably predicted as they depend on the localization mechanism and unknown parameters of extra-dimensional physics.

One may worry that non-conservation of electric charge in our world may be in contradiction with the fact that electrodynamics on the brane is effectively four dimensional. Indeed, the standard lore is that the charge conservation in our world is guaranteed by the four-dimensional Gauss' law (and causality, i.e. absence of action-at-a-distance). The purpose of this paper is to prove by example that there is no contradiction at all: in a model we consider, escape of charges into extra dimensions is perfectly consistent with localization of electromagnetism on the brane. The resulting picture is that physics becomes intrinsically multi-dimensional once the charged particles leave the brane; the four-dimensional Gauss' law is no longer valid; the electromagnetic field induced on the brane by escaping charges does not obey the four-dimensional Maxwell's equations; this induced field gradually decreases in a causal manner and finally disappears.

The issue we discuss in this paper has a close gravitational analogy: matter leaking into extra dimensions carries away energy, so one may wonder whether this is consistent with the "Gauss' law" of four-dimensional general relativity describing gravity on our brane. The gravitational field of escaping particles has been studied in ref. [15] with the results very similar to those outlined in the previous paragraph. We will closely follow ref. [15] both in spirit and in some of the technicalities.

To discuss the consistency of charge non-conservation on the brane with four-dimensional behavior of electromagnetism, we have in the first place to construct a model with a localized photon. A simple possibility is the localization of a photon due to gravity produced by the brane itself, which is very much the same mechanism as one leading to the localization of a graviton [9] and a scalar [16]. Consider space-time with $(4+n+1)$ dimensions, n of which are compact and one is infinite and "warped". For simplicity we consider compactification on n-dimensional torus. Let our world be a $(3+n)$-brane with n compact dimensions of very small size (smaller than TeV^{-1}). The brane has a certain tension, and there is a negative bulk cosmological constant. With this setup and appropriate fine-tuning between the bulk cosmological constant and brane tension, there exists a solution to $(4+n+1)$-dimensional Einstein equations with metric similar to the Randall-Sundrum [9] one,

$$ds^2 = a^2(z)\left[\eta_{\mu\nu}dx^\mu dx^\nu - \sum_{i=1}^{n} R_i^2 d\theta_i^2\right] - dz^2\,, \tag{1}$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski tensor, $\theta_i \in [0, 2\pi]$ are compact coordinates and R_i are (small) sizes of compact dimensions. Here

$$a(z) = \mathrm{e}^{-k|z|}$$

and k is determined by the bulk cosmological constant.

Let us now introduce the U(1) gauge field A_M propagating in the background (1). With appropriate rescaling, its free action is

$$S_{\text{gauge}} = -\frac{1}{4}\int dz \cdot \prod \frac{d\theta_i}{2\pi R_i} \cdot d^4x\sqrt{g}g^{MP}g^{NQ}F_{MN}F_{PQ}\,.$$

As we consider small R_i and low energies, we truncate this action to the zero Kaluza-Klein modes of compact n dimensions, i.e. take A_M independent of θ_i. To see that there exists a photon localized on the brane, consider the action for four-vector components $A_\mu(x,z)$ of the gauge potential,

$$S = -\frac{1}{4}\int dz a^n \eta^{\mu\nu}\eta^{\lambda\rho} F_{\mu\lambda}F_{\nu\rho} + \cdots .$$

We see that the measure determining the normalization factor is

$$\int dz a^n \equiv \int_{-\infty}^{\infty} dz \mathrm{e}^{-kn|z|} . \tag{2}$$

Hence, the z-independent mode, $A_\mu(x)$, is normalizable; this is the wave function of the localized photon, up to a normalization factor proportional to $\sqrt{kn}$. The reason of why we had to generalize the five-dimensional Randall-Sundrum setup is now obvious: at $n = 0$ there is no localized photon [16], but at $n \geq 1$ gauge fields localize on the brane, and electrodynamics on the brane becomes four dimensional at large distances.

The main purpose of this paper is to calculate the gauge field induced on the brane by charges escaping into the non-compact dimension. We again take the charged fields independent of θ_i. The motion of charges along the z-direction is then treated in the classical approximation. Let us consider for definiteness a single particle that has been at rest on the brane at $\mathbf{x} = 0$ and $t < 0$ and then (at time $t = 0$) escapes the brane towards positive z with zero initial velocity. (The same calculation applies to the process of escape of a particle in an orbifold setup with fixed point $z = 0$.)

The particle moves perpendicular to the brane and gets accelerated towards large z by the gravitational field [17, 15]. The world line of this particle is given by

$$\mathbf{x}_c = 0 ; \qquad z_c(t) = \frac{1}{2k}\ln(1 + k^2 t^2) . \tag{3}$$

For a particle of charge e, the non-vanishing components of the corresponding electromagnetic current are

$$\sqrt{g} j^0 = e\delta^{(3)}(\mathbf{x})\delta(z - z_c(t)) , \tag{4}$$

$$\sqrt{g} j^z = e\delta^{(3)}(\mathbf{x})\delta(z - z_c(t))\frac{dz_c}{dt} . \tag{5}$$

Hereafter $g = |\det g|$. This current induces the electromagnetic field everywhere in space-time according to Maxwell's equations

$$\partial_M \left(\sqrt{g} g^{MN} g^{PQ} F_{NQ}\right) = -\sqrt{g} j^P . \tag{6}$$

Since $j^{\theta_i} = 0$, it is consistent to set $A_{\theta_i} = 0$, and also consider A_M independent of θ_i. Furthermore, we can choose the gauge

$$A_z = 0 .$$

Then Maxwell's equations (6) reduce to two equations. On the right to the brane these are

$$\mathrm{e}^{-nkz}\partial_\mu F^{\mu\nu} - \partial_z\left(\mathrm{e}^{-(n+2)kz}\partial_z A^\nu\right) = -\delta^{\nu 0}\sqrt{g}j^0\,, \tag{7}$$
$$\mathrm{e}^{-(n+2)kz}\partial_z\partial_\mu A^\mu = -\sqrt{g}j^z\,, \tag{8}$$

where the four-dimensional indices are raised and lowered by the Minkowski metric.

As long as we are interested in the electromagnetic field *on the brane*, only eq. (7) is important. This equation (again on the right to the brane for definiteness) can be written in the following way:

$$\mathrm{e}^{-nkz}\left[\partial_\mu^2 - \mathrm{e}^{-2kz}\partial_z^2 + (n+2)k\mathrm{e}^{-2kz}\partial_z\right]A^\nu = -\delta^{\nu 0}\sqrt{g}j^0 + \mathrm{e}^{-nkz}\partial^\nu\left(\partial_\mu A^\mu\right). \tag{9}$$

The last term in this equation can be explicitly found by making use of eq. (8), but it produces pure gauge contribution *on the brane.* Indeed, the solution to eq. (9) is expressed in terms of the retarded Green's function, $G_R(x-x',z,z')$, of the operator entering the left-hand side

$$A^\nu(x,z) = -\int d^4x'dz'G_R(x-x',z,z')\delta^{\nu 0}\sqrt{g}j^0(x',z') +$$
$$+\,\partial_\nu\int d^4x'dz'G_R(x-x',z,z')\partial_\mu A^\mu(x',z')\mathrm{e}^{-nkz'}\,.$$

The second term here is pure gauge on the brane; it does not affect charges residing on the brane and hence can be omitted. The only relevant component of A^μ on the brane is then

$$A^0(x) \equiv A^0(x,z=0) = -\int d^4x'dz'G_R(x-x',0,z')\sqrt{g}j^0(x',z')\,.$$

Upon introducing a variable

$$\xi = \frac{1}{k}\mathrm{e}^{kz}$$

this expression is written in the following explicit form:

$$A^0(r,t) = -\int dt'\int_{1/k}^{\infty} d\xi' G_R\left(t-t',r,\xi=\frac{1}{k},\xi'\right)e\delta(\xi'-\xi_c(t'))\,. \tag{10}$$

We will be interested in length scales much larger than k^{-1}, so we will set

$$\xi_c(t) = t$$

in the rest of our analysis.

To proceed further, we need an explicit form of the retarded Green's function,

$$G_R(x-x',z,z') = \left[knD_0(x-x') + \int_0^\infty dm A_m(z)A_m(z')D_m(x-x')\right]. \tag{11}$$

Here the first term is the contribution of the zero mode, the second term comes from the continuum of non-zero modes. We can divide the continuum modes into symmetric and anti-symmetric subsets with respect to the brane. Anti-symmetric modes vanish on the brane, so they do not contribute to the Green's function at $z = 0$. Since we are calculating the gauge field on the brane, we can neglect anti-symmetric modes in eq. (11). The symmetric modes are

$$A_m(z) = \mathrm{e}^{k\nu z}\sqrt{\frac{m}{2k}}\,\frac{N_{\nu-1}\left(\frac{m}{k}\right) J_\nu\left(\frac{m}{k}\mathrm{e}^{kz}\right) - J_{\nu-1}\left(\frac{m}{k}\right) N_\nu\left(\frac{m}{k}\mathrm{e}^{kz}\right)}{\sqrt{N_{\nu-1}\left(\frac{m}{k}\right)^2 + J_{\nu-1}\left(\frac{m}{k}\right)^2}}\,, \tag{12}$$

where

$$\nu = \frac{n}{2} + 1\,.$$

These continuum modes are normalized to $\delta(m - m')$ with the weight (2). The retarded four-dimensional massive propagators entering eq. (11) are conveniently written in coordinate representation,

$$\begin{aligned} D_m(x) &= -\frac{1}{2\pi}\theta(t)\delta(\lambda^2) + \frac{m}{4\pi\lambda}\theta(t - |\mathbf{x}|)J_1(m\lambda)\,, \\ \lambda &= \sqrt{t^2 - |\mathbf{x}|^2}\,. \end{aligned} \tag{13}$$

As a consistency check of our calculation, we point out that had one boldly made use of the zero mode approximation, i.e. neglected the continuum contribution in eq. (11), one would obtain from eq. (10) that the induced gauge potential A^0 is a static Coulomb potential, $A^0 \propto 1/r$. This is not surprising, as electrodynamics is effectively four dimensional in the zero mode approximation.

The zero mode approximation is not justified when charges move outside the brane, so we proceed to evaluate the complete Green's function. At large distances, $t, |\mathbf{x}| \gg k^{-1}$, and well away from the light cone, $\lambda \gg k^{-1}$, only relatively small masses m are relevant, and we obtain

$$G_R(x, 0, z') = -\frac{\theta(t - |\mathbf{x}|)}{(2k)^{\nu-1}4\pi\lambda\Gamma(\nu - 1)}\mathrm{e}^{k\nu z'}\int_0^\infty dm m^\nu J_1(m\lambda)J_\nu(m\xi')\,, \tag{14}$$

where we set $z = 0$, as we are interested in the induced field on the brane. Note that the first term in eq. (13) cancels out due to completeness of the set of the modes (including the zero mode).

The induced electromagnetic potential is obtained by plugging the expression for the Green's function, eq. (14), into eq. (10). The evaluation of the resulting integrals is somewhat cumbersome, and we outline this step in appendix A. Let us present the results for $A^0(\mathbf{x}, t) \equiv A^0(r, t)$ for $n = 1$ and $n = 2$.

$n = 1$:

$$A^0 = \frac{ke}{4\pi^2}\left[\frac{\sqrt{t^2 - r^2}}{t^2} + \frac{1}{r}\left\{\mathrm{arctg}\left(\frac{t + r}{t - r}\right)^{1/2} - \mathrm{arctg}\left(\frac{t - r}{t + r}\right)^{1/2}\right\}\right]. \tag{15}$$

JHEP08(2000)041

$n = 2:$

$$A^0 = \frac{ek}{4\pi} \cdot \frac{3t^2 - r^2}{2t^3} . \tag{16}$$

These expressions are valid inside the light cone $r < t$, whereas outside the light cone A^0 is still equal to the Coulomb potential generated by the charge that has been at rest on the brane at negative times,

$$A^0 = \frac{nek}{8\pi} \cdot \frac{1}{r} ,$$

the factor $nk/2$ coming from the normalization of the zero photon mode. At both $n = 1$ and $n = 2$, the gauge potential A^0 and electric field $E = -\partial_r A^0$ are continuous on the light cone. Deep inside the light cone, i.e. at $r \ll t$, the electric field gradually disappears,

$$E \propto \frac{r}{t^3} . \tag{17}$$

The case of arbitrary even n is also possible to treat analytically; by making use of eq. (A.5) we have checked that the same properties (continuity of A^0 and E across the light cone and the behavior (17) at $r \ll t$) hold for all even n. The electromagnetic field induced on the brane by escaping charges switches off in a causal way.

We conclude that the electric charge non-conservation in the brane world via the leakage of charged particles into extra dimensions is consistent with effectively four-dimensional electrodynamics governing the long distance interactions of charges residing on the brane. In the setup we introduced in this paper, escaping charge induces spherical electromagnetic wave on the brane; beyond this wave electromagnetic field gradually disappears. This wave is a collective effect of the photon zero mode and continuum modes.

The existence of continuum bulk modes with arbitrarily small four-dimensional "masses" is crucial for this phenomenon, as it makes the problem intrinsically five dimensional even at large distances, and in this way the 4-d Gauss' law obstruction to the electric charge non-conservation is avoided. We expect that the non-conservation of electric charge on the brane is possible in all models with localized photon and continuum of electromagnetic modes in the bulk that starts from zero four-dimensional "mass". These properties are inherent, in particular, in the six-dimensional string setup of Gherghetta and Shaposhnikov [18]; the localization of a photon in the latter setup has been discussed recently by Oda [19]. On the other hand, those mechanisms of the photon localization which do not incorporate the bulk continuum starting from zero will not allow for the electric charge non-conservation in the brane world; an example is the Dvali-Shifman setup [8]. It remains to be understood which possibility is preferable from the point of view of string/M-theory.

May colored particles — quarks — escape from our brane too? Certainly not in theories with confinement of color both on the brane and in the bulk. However, if color is not confined in the bulk, one may speculate on the following possibility.

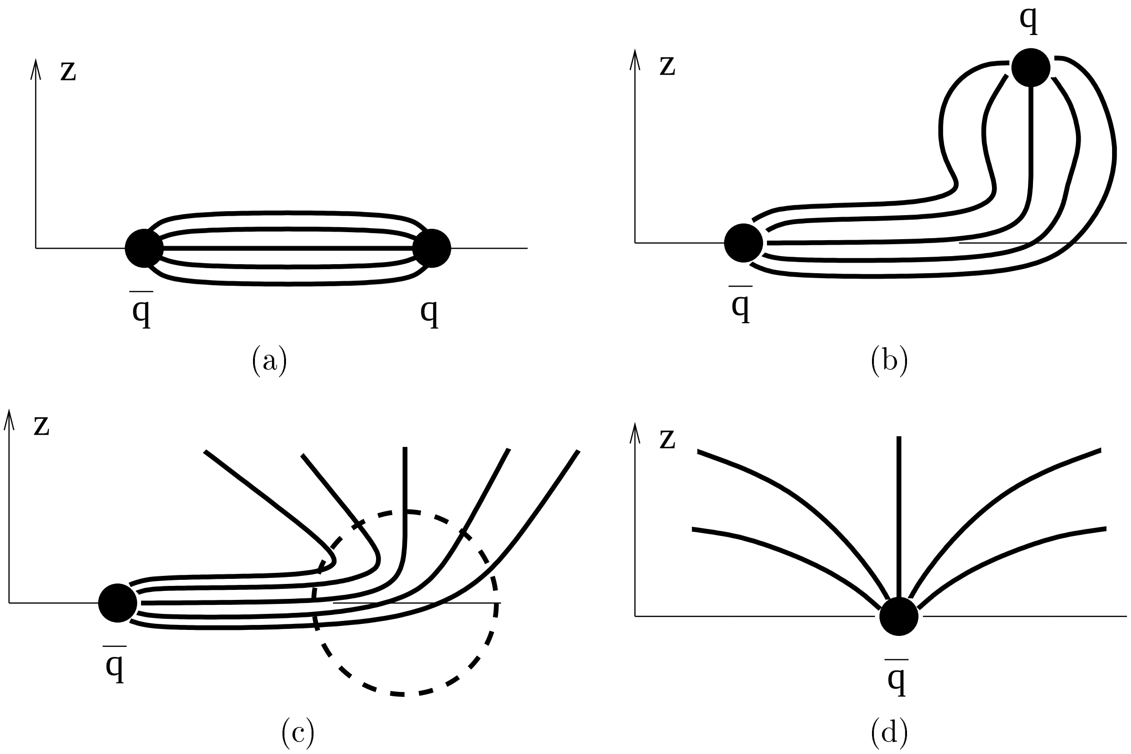

Figure 1: Formation of colorless anti-quark state.

Consider a well separated pair of quark and anti-quark on the brane, with color flux tube stretching between them (figure $1a$). Suppose that the quark leaves the brane (figure $1b$); if color is not confined in the bulk, this does not cost very large energy even if the quark travels far away from the brane. After the quark has moved away, there remains an object on the brane (shown by dashed line in figure $1c$) which, from the four-dimensional point of view, behaves as a color triplet state with zero baryon number, electric and weak charges, etc. This object may combine with anti-quark to form a free anti-quark colorless state (again as viewed from four dimensions) with quantum numbers of anti-quark (figure $1d$). It is worth trying to undertsand at a more quantitative level whether these exotic objects may indeed exist in models with infinite extra dimensions.

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A. Calculation of the electromagnetic potential

The calculations of the integral entering eq. (14) are different for even n (integer ν) and odd n (half-integer ν). Let us begin with even n. One makes use of the relation

$$m^\nu J_\nu(m\xi) = (-1)^\nu \xi^\nu \left(\frac{d}{\xi \partial \xi}\right)^\nu J_0(m\xi)$$

and obtains

$$\begin{aligned}\int_0^\infty dm m^\nu J_1(m\lambda) J_\nu(m\xi) &= (-1)^\nu \xi^\nu \left(\frac{d}{\xi\partial\xi}\right)^\nu \int_0^\infty dm J_1(m\lambda) J_0(m\xi) \\ &= (-1)^\nu \xi^\nu \left(\frac{d}{\xi\partial\xi}\right)^\nu \frac{\theta(\lambda-\xi)}{\lambda}\,. \end{aligned} \tag{A.1}$$

Hence, the Green's function is concentrated on the five-dimensional light cone $\xi = \lambda$. Plugging this expression into eq. (14) and then eq. (10) one finds that the induced field on the brane is given by the following integral:

$$\begin{aligned} A^0 &= -\alpha_e \int dt' \int_{1/k}^\infty d\xi (k\xi)^n \frac{\theta(t-t'-r)}{k^{2\nu-3}\lambda(t')^2} \frac{\delta\left(t' - \sqrt{\xi^2 - k^{-2}}\right)}{\sqrt{1-(k\xi)^{-2}}} \times \\ &\quad \times \xi^2 \left(\frac{d}{\xi d\xi}\right)^\nu \theta\left(\lambda(t') - \xi\right), \end{aligned} \tag{A.2}$$

where

$$\alpha_e = \left(-\frac{1}{2}\right)^{\nu-1} \frac{e}{4\pi\Gamma(\nu-1)}$$

and

$$\lambda(t')^2 = (t-t')^2 - r^2\,.$$

We further simplify this integral by making use of the identity

$$\frac{d}{\xi d\xi} f(\xi) = 2\frac{d}{d\beta} f\left(\sqrt{\xi^2+\beta}\right)\Big|_{\beta=0} \tag{A.3}$$

which holds for arbitrary function $f(\xi)$. Then eq. (A.2) takes the form

$$\begin{aligned} A^0 &= \alpha_e k \left(2\frac{d}{d\beta}\right)^{\nu-1} \int dt' \int_0^\infty d\xi \xi^{n+2} \frac{\theta(t-t'-r)}{\lambda(t')^2} \times \\ &\quad \times \frac{\delta(t'-\xi)}{\sqrt{\xi^2+\beta}} \delta\left(\lambda(t') - \sqrt{\xi^2+\beta}\right)\Big|_{\beta=0}, \end{aligned} \tag{A.4}$$

where subleading in λ/k terms have been omitted. Now it is straightforward to perform the integration with the result

$$A^0(r,t) = \alpha_e k \left(2\frac{d}{d\beta}\right)^{\nu-1} \frac{\xi_*^{n+2}}{t(\xi_*^2+\beta)}\Bigg|_{\beta=0}, \tag{A.5}$$

where

$$\xi_* = \frac{t^2 - r^2 - \beta}{2t} \,. \tag{A.6}$$

At $n = 2$ eq. (A.5) reduces to eq. (16).

Let us now evaluate the integral entering eq. (11) at odd n (half-integer ν). We make use of the relation

$$m^{l+1/2} J_{l+1/2}(m\xi) = (-1)^l \sqrt{\frac{2}{\pi}} \xi^{l+1/2} \left(\frac{d}{\xi \partial \xi} \right)^l \frac{\sin m\xi}{\xi} \,.$$

Then

$$\begin{aligned} \int_0^\infty dm m^\nu J_1(m\lambda) J_\nu(m\xi') &= (-1)^l \sqrt{\frac{2}{\pi}} \xi^\nu \left(\frac{d}{\xi \partial \xi} \right)^l \int_0^\infty dm J_1(m\lambda) \frac{\sin m\xi}{\xi} \\ &= (-1)^l \sqrt{\frac{2}{\pi}} \xi^\nu \left(\frac{d}{\xi \partial \xi} \right)^l \frac{\theta(\lambda - \xi)}{\lambda \sqrt{\lambda^2 - \xi^2}} \,. \end{aligned} \tag{A.7}$$

This expression is again plugged into eq. (14) and then eq. (10). The integration over t' is performed by making use of the trick (A.3), and we find

$$A^0 = \alpha_o k \left(2 \frac{d}{d\beta} \right)^{\nu - 1/2} \int_0^{\xi_*} \frac{d\xi \xi^{n+2}}{\lambda(\xi)^2 \sqrt{\lambda(\xi)^2 - \xi^2 - \beta}} \Bigg|_{\beta = 0} \,, \tag{A.8}$$

where

$$\alpha_o = \left(-\frac{1}{2} \right)^{\nu - 3/2} \frac{e}{4\pi^{3/2} \Gamma(\nu - 1)} \,.$$

At $n = 1$ the integral (A.8) can be evaluated explicitly, and one obtains eq. (15).

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